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THE STABILITY OF A PLANE LONGITUDINAL SHOCK WAVE IN AN ISOTROPIC ELASTIC MEDIUM†

S. A. YEGORYUSHKIN and G. V. SHUBIN

Moscow

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The problem of the linear stability of fast plane longitudinal shock waves (SW) in an isotropic elastic body whose elastic potential is a given function of the deformation tensor invariants depending additively on entropy is considered. When the medium is in a state of uniaxial compression or extension, the resulting dispersion equation can be factorized. Assuming that ahead of the SW the medium is in a state of uniaxial compression (extension), sufficient conditions for the instability of longitudinal SWs are obtained. When the medium is in a state of uniaxial extension ahead of the SW and the velocity of the SW is such that the deformations behind the SW are close to zero and much smaller than those ahead of the SW, the problem of linear stability is solved completely, i.e. the necessary and sufficient conditions for stable, unstable and neutrally stable SWs to exist are stated. All the results obtained remain valid in the case of a medium with transverse anisotropy (the direction of the anisotropy axis coinciding with the direction of SW propagation), and also for an isotropic medium in a state of compression (extension) in the direction of two mutually perpendicular axes lying in the plane perpendicular to the direction of SW propagation, the deformations along these axes being equal. Copyright © 1996 Elsevier Science Ltd.

1. DERIVATION OF AN EQUATION FOR THE NATURAL FREQUENCIES

Consider a plane longitudinal shock wave (SW) propagating at constant Lagrangian velocity W_0 with respect to the particles of an isotropic elastic medium, the elastic potential Φ of which is a function of the invariants I_k of the strain tensor ε_{ij} which depends additively on the entropy. The last assumption holds for SWs of moderate intensity and corresponds to the universally adopted [1] expansion of Φ in powers of I_k ($I_1 = \varepsilon_{ii}$, $I_2 = \varepsilon_{ik}\varepsilon_{ik}$, $I_3 = \varepsilon_{ij}\varepsilon_{jk}\varepsilon_{ki}$) up to and including the fourth power of ε_{ij}. Let W_0 be perpendicular to the SW plane and directed along the ξ_3 axis of a rectangular Cartesian system of coordinates (ξ_1, ξ_2, ξ_3). Among the components of the strain tensor in the domain ahead of the SW only ε_{33} may be non-zero and is assumed to be constant. The system of coordinates (ξ_1, ξ_2, ξ_3) initially coincides with the Lagrangian system behind the SW. The propagation of the shock wave in the medium results in compression along the ξ_3 axis, i.e. the directions of the axes of the Lagrangian system of coordinates and the chosen system of coordinates (ξ_1, ξ_2, ξ_3) coincide ahead of the SW front as well as behind it.

The equations of motion of the medium described above can be written in the form [1]

$$\rho_0 \frac{\partial V_j^n}{\partial t} = \frac{\partial^2 \Phi}{\partial \xi_i \partial U_{ij}}, \quad \frac{\partial U_{ij}^n}{\partial t} = \frac{\partial V_i^n}{\partial \xi_j}, \quad i,j = 1,\ldots,3; \quad n = 1,2 \tag{1.1}$$

Here $V^n = \{V_1^n, V_2^n, V_3^n\}$ is the velocity vector of the medium, U_{ij}^n are the components of the tensor of displacement gradients, ρ_0 is the density, and the superscript corresponds to the states ahead of the SW ($n = 1$) and behind it ($n = 2$).

Equations (1.1) form a system of first-order quasilinear hyperbolic equations and can have solutions containing surfaces of strong discontinuity, on which the conditions

$$W[U_{ij}] + [V_i]n_j = 0 \tag{1.2}$$

$$\rho_0 W[V_i] + [\partial\Phi / \partial U_{ij}]n_j = 0 \quad ([f] = f^2 - f^1)$$

are satisfied ($\mathbf{n} = \{n_1, n_2, n_3\}$ is the vector of the normal to the surface of discontinuity and W is the velocity of the discontinuity).

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For a plane SW propagating at constant velocity W_0 along the ξ_3 axis conditions (1.2) lead to the relations

$$\rho_0 W_0^2 [U_{i3}] = [\partial\Phi / \partial U_{i3}] \tag{1.3}$$

connecting the strain states ahead of the SW and behind it. It follows from (1.3) that if $U_{i3}^{(1)} \neq 0$ only for $i = 3$, then a solution exists for which $U_{33}^{(2)}$ will also be the only non-zero component of the displacement gradient tensor behind the SW.

To study the stability of the SW we change to a system of coordinates $\{X_1, X_2, X_3\}$ moving with the SW so that $X_1 = \xi_1, X_2 = \xi_2, X_3 = \xi_3 - W_0 t$. Then the domain $X_3 > 0$ corresponds to the state ahead of the SW and the domain $X_3 < 0$ corresponds to the state behind the SW.

We will consider weak perturbations of the SW surface described by the equation $X_3 - \varepsilon\chi(X_1, X_2, t) = 0, \varepsilon \ll 1$, where $\chi = \chi'_0 \exp(-i\Omega t + ik_1X_1 + ik_2X_2)$. Then the perturbations of the physical quantities can be sought in the form

$$\mathbf{U} = (\mathbf{V}', \mathbf{U}') = (\mathbf{V}'_0, \mathbf{U}'_0)\exp(-i\Omega t + i(\mathbf{kX})) \tag{1.4}$$

where $\mathbf{k} = \{k_1, k_2, k_3\}$ is the wave vector and $\mathbf{X} = (X_1, X_2, X_3)$. When solving the problem it is sufficient to restrict oneself to perturbations of this form only [2].

Because of the evolutionary character of the fast discontinuity under consideration the perturbations of the physical quantities in the domain ahead of the SW are incoming and can be assumed to be absent. In the domain behind the SW the small perturbations satisfy relations which can be obtained by linearizing system (1.1) after writing it down in the moving coordinates $\{X_1, X_2, X_3\}$. Taking (2.4) into account, we can write these relations in the form

$$-\omega U'_{ij} = k_j V'_i, \quad -\rho_0 \omega V'_j = k_l A_{ljsp} U'_{sp} \tag{1.5}$$

$$A_{ljsp} = \partial^2\Phi / \partial U_{lj} \partial U_{sp}, \quad \omega = \Omega + W_0 k_3$$

The system of linear equations (1.5) has a non-trivial solution if and only if its determinant is zero. This condition leads to a relation between the frequency Ω and the components k_j of the wave vector $\mathbf{k}$

$$[(A_{3333}k_3^2 + A_{3131}k_\tau^2 - \rho_0\omega^2)(A_{1313}k_3^2 + A_{1111}k_\tau^2 - \rho_0\omega^2) -$$
$$-c_0^2 k_3^2 k_\tau^2](A_{1313}k_3^2 + A_{1212}k_\tau^2 - \rho_0\omega^2) = 0 \tag{1.6}$$
$$c_0 = A_{1331} + A_{1133}$$

We emphasize that the coefficients of the dispersion equation (1.6) depend only on k_τ^2 and are independent of the components k_1 and k_2 of the wave vector taken on their own. This is due to the absence of transverse deformations in the domain behind the SW, as a result of which all directions tangent to the SW plane are equivalent and the velocities of propagation of weak perturbations depend only on the angle between the wave vector $\mathbf{k}$ and the X_3 axis.

The quantities ($\mathbf{V}'$, $\mathbf{U}'$) satisfy linear boundary conditions on the SW, which can be obtained after substituting the expansions

$$W = W_0 + \varepsilon\frac{\partial\chi}{\partial t}, \quad U_{ij} = U_{ij}^{(2)} + \varepsilon U'_{ij}$$

$$V_j = V_j^{(2)} + \varepsilon V'_j, \quad n = \left\{-\varepsilon\frac{\partial\chi}{\partial x_1}, \ -\varepsilon\frac{\partial\chi}{\partial x_2}, \ 1\right\}$$

into (1.2) and retaining terms of order ε using (1.3). Taking (1.4) into account, we obtain the system

$$-2\rho_0 W_0 i\Omega\chi'_0[U_{33}^0] = U'_{33}[\overset{<2>}{A}_{3333} - \rho_0 W_0^2] + \overset{<2>}{A}_{3311}U'_{11} + \overset{<2>}{A}_{3322}U'_{22}$$

$$\rho_0 W_0^2 U'_{j3} = -[\partial\Phi / \partial U_{jj}]ik_j\chi'_0 + \overset{<2>}{A}_{j3j3}U'_{j3} + \overset{<2>}{A}_{j33j}U'_{3j}$$

$$W_0U'_{j3} + V'_i = 0, \quad W_0U'_{33} - i\Omega\chi'_0[U^0_{33}] + V'_3 = 0 \tag{1.7}$$

$$U'_{1j} = U'_{2j} = 0, \quad U'_{3j} + ik_j\chi'_0[U^0_{33}] = 0, \quad j = 1,2$$

of 12 equations relating the values of the 13 unknowns U'_{ij}, V'_j, χ'_0 on the SW. To close the system we use the following fact. The dispersion equation (1.6) has six roots $k_{3n} \equiv k_{3n}$ (Ω, k_τ^2), $(n = 1, \ldots, 6)$ corresponding to longitudinal and transverse sound waves, of which only one longitudinal wave arrives at the discontinuity because of the evolutionary nature of the SW (since the SW under consideration is a fast one, there are no perturbations ahead of it).

We assume that U'_{ij} and V'_j are bounded as $X_3 \to -\infty$ and only the surface of discontinuity is perturbed at $t = 0$, the solution perturbations in the domains ahead of the discontinuity and behind it being equal to zero. Because of this, a relation [3] excluding incoming perturbations must hold on the SW [3]. To study this relation we write the linearized original system (1.1) in the form

$$\mathbf{E}\frac{\partial \mathbf{U}}{\partial t} - \frac{1}{\rho_0}\left(\mathbf{H}\frac{\partial \mathbf{U}}{\partial x_3} + \mathbf{F}\frac{\partial \mathbf{U}}{\partial x_2} + \mathbf{G}\frac{\partial \mathbf{U}}{\partial x_1}\right) = 0 \tag{1.8}$$

(**U** is a vector formed by the components of the vector **V** and the tensor **U**).

We apply a Laplace transformation with respect to t and X_3 and a Fourier transformation with respect to X_1 and X_2 to (1.8). Then, using the boundary conditions on the SW and finding the inverse Laplace transform, we obtain

$$\mathbf{U}(x_3) = \frac{1}{2\pi i}\int \frac{\mathbf{H}\hat{\mathbf{U}}_0(\Omega, k_3, k_2, k_1)}{i(k_3\mathbf{H} + k_2\mathbf{F} + k_1\mathbf{G} + \rho_0\Omega\mathbf{E})} e^{-ik_3x_3}dk_3 \tag{1.9}$$

where $\hat{\mathbf{U}}_0$ is the value of $\hat{\mathbf{U}}$ on the SW.

Solution (1.9) is determined by the poles of the integrand. But the incoming waves cannot make a contribution to the solution, since U_{ij} and V_j are bounded as $X_3 \to \infty$.

Let **L** be the left eigenvector corresponding to an incoming longitudinal wave, i.e. $(\mathbf{L}\,(k_3\mathbf{H} + k_2\mathbf{F} + k_1\mathbf{G} + \rho_0\Omega\mathbf{E}) = 0)$. Solution (1.9) will exist if

$$(\mathbf{H}\hat{\mathbf{U}}_0, \mathbf{L}) = 0 \tag{1.10}$$

This is the additional relationship which closes system (1.7). For a non-trivial solution of system (1.7), (1.10) to exist its determinant must be zero. This condition reduces to a relationship from which we can determine the natural frequency Ω. In this way one can solve the problem of the stability of the SW. When $c_0 \neq 0$, the condition has the form

$$D(\Omega, k_3, k_\tau^2) \equiv c_1\rho_0\omega^2 + c_2k_\tau^2 + c_3k_3^2 = 0 \tag{1.11}$$

$$c_1 = -(A_{1133}[U^0_{33}] - [\partial\Phi/\partial U_{11}]), \quad c_2 = -A_{1313}c_1$$

$$c_3 = c_0d_3[U^0_{33}] - A_{3333}c_1, \quad d_j = (W_0^2\rho_0 - A_{j3j3})$$

(the values of A_{ijsq} are taken behind the SW front).

But if $c_0 = 0$, the condition that the determinant of system (1.7), (1.10) must equal zero reduces to $\omega k_3[U^0_{33}]d_3 = 0$. This means that the point on the shock adiabatic curve corresponding to such a discontinuity is a Jouguet point. We shall therefore henceforth assume that the condition $c_0 = 0$ is violated.

If Eq. (1.11) has a root Ω whose imaginary part is positive, the perturbations increase with time and the SW is unstable. When $\operatorname{Im}\Omega < 0$, the SW is stable. But if $\operatorname{Im}\Omega = 0$, the perturbations remain bounded and a neutral stability regime occurs. The difficulty in solving Eq. (1.11) lies in the fact that the branch $k_3 \equiv k_3$ (Ω, k_τ^2) of the dispersion equation appearing in it is a rather complicated algebraic function of the complex variable Ω.

2. ASYMPTOTIC PROPERTIES OF THE EQUATION FOR THE NATURAL FREQUENCIES

The complete system of independent dimensionless complexes $\{\gamma_1, \gamma_2, \dots, \gamma_n\}$ formed by the physical constants which occur in the problem will be called the space of determining parameters for problem (1.5), (1.7). The space $\{\gamma_1, \gamma_2, \dots, \gamma_n\}$ can be represented as a direct sum of subdomains of dimensions N such that the solution of problem (1.5), (1.7) is stable, unstable or neutrally stable at internal points of each of the subdomains [2]. The solution alters its asymptotic behaviour on the surfaces $G_\alpha(\gamma_1, \gamma_2, \dots, \gamma_n) = 0$ serving as the boundaries of these subdomains.

It was shown in [2] that if the components k_1 and k_2 of the wave vector appear in D only through $k_\tau^2 = k_1^2 + k_2^2$, then one of the boundaries G_α of the instability domain corresponds to a transfer of a pair of real roots Ω of D onto the imaginary axis $\operatorname{Re}\Omega = 0$ through infinity. (For example, in gas dynamics this boundary is given by $\delta = -1 - 2M$, where δ is the dimensionless derivative along the shock adiabatic curve and M is the Mach number behind the SW [4].)

Indeed, as $\Omega \to \infty$ the leading term of $k_3 = k_3(\Omega, k_\tau^2)$ with respect to $| k_\tau/\Omega |$ becomes linear and $D(\Omega, k_\tau^2)$ becomes a polynomial

$$D(\Omega, k_\tau^2) = b_0\Omega^2 + b_1 k_\tau^2 + O(k_\tau^2 / \Omega^2) = 0 \tag{2.1}$$

As $b_0 \to \infty$ one of the pairs of roots of D has the asymptotic form

$$\Omega = \pm(-b_1 / b_0)^{1/2} |k_\tau|$$

which corresponds to the occurrence of instability described above if the surface $b_0(\gamma_1, \gamma_2, \dots, \gamma_n) = 0$ is taken as G_α. It follows that $b_0(\gamma_1, \gamma_2, \dots, \gamma_n) = 0$ is a sufficient condition for the SW to be unstable.

To construct G_α we observe that the leading term with respect to k_τ/Ω in the problem of the propagation of small perturbations becomes one-dimensional as $\Omega \to \infty$. Therefore, the following relation holds for the longitudinal sound wave arriving at the discontinuity

$$k_3 = \Lambda_1 - \Lambda_2 \frac{k_\tau^2}{\Omega^2}$$

$$\Lambda_1 = \frac{\Omega}{a_0 - W_0}, \quad \Lambda_2 = \frac{\{c_0^2 - A_{1313}(A_{3333} - A_{1313})\}}{2(A_{3333} - A_{1313})A_{3333}} a_0, \quad a_0 = \left(\frac{A_{3333}}{\rho_0}\right)^{1/2}$$

Substituting this asymptotic form into (1.11), we obtain

$$b_0 = c_0 d_3 [U_{33}^0] / (a_0 - W_0)^2$$

$$b_1 = A_{1313}\left(A_{1133}[U_{33}^0] - \left[\frac{\partial\Phi}{\partial U_{11}}\right]\right) - 2\Lambda_1\Lambda_2 d_3\left(A_{1331}[U_{33}^0] - \left[\frac{\partial\Phi}{\partial U_{11}}\right]\right)$$

(the values of A_{ijsq} are taken behind the SW front).

It is clear that the condition $b_0 = 0$ is violated under the assumptions made. Therefore, there is no surface corresponding to a transition to instability as $\Omega \to 0$.

We will show that a surface on which a transition to instability occurs may exist in the space of the governing parameters of the problem as $\Omega \to 0$.

When $\Omega \to \infty$ the solution of the dispersion equation (1.6) can be represented in the form

$$k_3 = k_\tau(m_1 + m_2\Omega / k_\tau + O(\Omega^2 / k_\tau^2)) \tag{2.2}$$

where

$$m_1^2 = (b + \sqrt{b^2 - 4ac}) / (2a)$$

$$a = d_1 d_3, \quad b = A_{1111}d_3 + A_{1313}d_1 + c_0^2, \quad c = A_{1111}A_{1313}$$

correspond to an incoming wave.

Since $a < 0$ and $c > 0$, it is always true that $m_1^2 < 0$. This means that the system of linearized equations of elasticity lacks hyperbolicity for small Ω. Substituting (2.2) into (1.11), we obtain

$$D = s_0 + s_1\Omega / k_\tau + O(\Omega^2 / k_\tau^2) \tag{2.3}$$

$$s_0 = A_{1313}\left(A_{1133}[U_{33}^0] - \left[\frac{\partial\Phi}{\partial U_{11}}\right]\right) + m_1^2 d_3\left(A_{1331}[U_{33}^0] + \left[\frac{\partial\Phi}{\partial U_{11}}\right]\right)$$

$$s_1 = 2m_1\rho_0 W_0\left(\left[\frac{\partial\Phi}{\partial U_{11}}\right] - A_{1133}[U_{33}^0]\right) + m_2 d_3\left(A_{1331}[U_{33}^0] + \left[\frac{\partial\Phi}{\partial U_{11}}\right]\right)$$

It follows from (2.3) that $\Omega = -s_0 s_1^{-1} k_1$. Since m_1 is imaginary, we have $s_1 = s_{11} + is_{12}$, $s_{12} \neq 0$ and a transition to instability occurs on the surface $s_0 = 0$, with

$$\operatorname{Im}\Omega = \operatorname{Im}\left(-\frac{s_0}{s_1}k_\tau\right) = k_\tau \frac{s_{12}s_0}{s_{11}^2 + s_{12}^2}$$

Therefore instability is guaranteed on the side of the surface $s_0 = 0$ where $s_{12}s_0 > 0$.

3. ON THE STABILITY OF A SW OF SPECIAL FORM

Consider SWs whose velocity of propagation W_z is related in a certain way to U_{33}. We assume that the state of the medium behind the SW, determined from (1.3), is undeformed, i.e.

$$U_{33}^{(2)} \equiv U_{33}^{(2)}(U_{33}^{(1)}, W_z) = 0 \quad (\partial\Phi / \partial U_{ij})^{(2)} = 0 \tag{3.1}$$

Moreover

$$W_z = W_z(U_{33}^{(1)}) = \left[\frac{1}{\rho_0 U_{33}^{(1)}}\left(\frac{\partial\Phi}{\partial U_{33}}\right)^{(1)}\right]^{1/2} \tag{3.2}$$

Therefore, for a given initial value of $U_{33}^{(1)}$ there is an evolutionary SW that satisfies (3.1), if W_z satisfy the evolution conditions (or Lax inequalities)

$$A_{1313}^{(2)} < \frac{1}{U_{33}^{(1)}}\left(\frac{\partial\Phi}{\partial U_{33}}\right)^{(1)} < A_{3333}^{(2)}, \quad \frac{1}{U_{33}^{(1)}}\left(\frac{\partial\Phi}{\partial U_{33}}\right)^{(1)} > A_{3333}^{(1)} \tag{3.3}$$

The second inequality in (3.3) implies, in particular, that the expression under the root sign in (3.2) is greater than zero.

When $U_{33}^{(2)} = 0$, all directions in the medium behind the SW front are equivalent. The dispersion equations (1.6) can be simplified, taking the form

$$(k_\tau^2 + k_3^2 - \alpha_1^2)^2(k_\tau^2 + k_3^2 - \alpha_3^2) = 0, \quad \alpha_j^2 = \rho_0\omega^2 / A_{j3j3} \tag{3.4}$$

The dispersion equation (3.4) has six roots, four of which ($k_\tau^2 + k_3^2 - \alpha_1^2$) correspond to transverse waves and two ($k_\tau^2 + k_3^2 - \alpha_3^2$) to longitudinal waves. The last relation implies that

$$k_3(\Omega_0) = \frac{\Omega_0 M^2 + [\Omega_0^2 M^2 - (1 - M^2)k_\tau^2]^{1/2}}{1 - M^2} \tag{3.5}$$

$$\Omega_0 = \frac{\Omega}{W_z}, \quad M^2 = \frac{\rho_0 W_z^2}{A_{3333}} < 1$$

in the incoming longitudinal waves.

Substituting (3.5) into (1.11) and taking into account the fact that

$$A_{1331}+A_{1133}=A_{3333}-A_{1313},\quad A_{1111}=A_{3333}$$

when $U_{33}^{(2)}=0$, we obtain

$$D(\Omega_0)=\Omega_0^2(1+M^2)+2\Omega_0[\Omega_0^2M^2-(1-M^2)k_\tau^2]^{1/2}-\sigma(1-M^2)k_\tau^2=0 \tag{3.6}$$

$$\sigma=[(\partial\Phi/\partial U_{11})/U_{33}^1-A_{1133}+A_{3333}]/(\rho_0W_z^2)$$

where $\sigma\approx 1$ for waves of low intensity.

It was shown in [2] that the boundaries between different stability states correspond to the following cases.

1. The boundary between neutral stability and instability occurs when the root of (3.6) passes through infinity. No boundary between states (see above) occurs.

2. The boundary between neutral stability and stability occurs when the branching point of $k_3(\Omega_0)$ is a root of $D(\Omega, k)$. The equation of this surface in the space of governing parameters will be

$$1+M^2-\sigma M^2=0 \tag{3.7}$$

If $1+M^2-\sigma M^2>0$, the SW is stable. Otherwise it is neutrally stable. This inequality holds for SWs of low intensity.

3. The transition from stability to instability corresponds to the appearance of multiple roots of $D(\Omega, k_\tau)$ on the real axis or a root falling outside the cut between the branch points of $k_3(\Omega)$. The latter possibility occurs at the point $\Omega=0$. The value $\Omega=0$ is a root of $D(\Omega, k_\tau)$ if $\sigma=0$. This case is not realized for waves of low intensity, since $\sigma\approx 1$ for such waves.

Thus, if $\sigma<0$, the SW will be unstable, if $0<\sigma<(1+M^2)/M^2$, it will be stable, and if $\sigma>(1+M^2)/M^2$, it will be neutrally stable. It follows from what was said above that SWs of low intensity are always stable. Various states (depending on σ) can be realized using the model of SWs of finite intensity considered in this paper.

For example, if the medium is given by the potential

$$\Phi=2\mu I_1^2+\mu I_2+\frac{1408}{147}\mu I_1^3+40\mu I_2^2$$

and the state in front of the wave is given by $U_{33}^{(1)}=-1/4$, the state behind the SW front will be $U_{33}^{(2)}=0$ if the SW passes with velocity W_z given by (3.2). In this case $\sigma=0$, and the evolution conditions are satisfied, and so are the conditions requiring that the squares of the characteristic velocities must be positive. If $U_{33}^{(1)}=-0.24$, then $0<\sigma<(1+M^2)/M^2$, and the SW is stable. If $U_{33}^{(1)}=-0.26$, then $\sigma<0$, and the SW is unstable.

If the medium is given by the potential $\Phi=0.1\mu I_1^2+\mu I_2-0.1\mu I_1^3-0.2\mu I_2^2$, and the state ahead of the SW is given by $U_{33}^{(1)}=-0.3$, then the state behind the SW front will be $U_{33}^{(2)}=0$ if the SW passes with velocity W_z determined by (2.2). In this case $\sigma>(1+M^2)/M^2$, and the SW will be neutrally stable.

In gas dynamics the equation for perturbation frequencies has the form

$$\Omega^2(1+\delta)+2\Omega[\Omega^2M^2-(1-M^2)k_\tau^2]^{1/2}-\sigma(\delta-1)k_\tau^2=0 \tag{3.8}$$

where $\sigma=\rho_2/\rho_1>1$ is the ratio of the density behind the jump and that ahead of the jump, δ is the dimensionless derivative along the shock adiabatic curve, and M is the Mach number behind the SW.

Unlike Eq. (3.6), the presence of the additional parameter δ is related to entropy in gas dynamics. If a medium with internal energy of the form

$$E(v,s)=E_1(v)+E_2(s)+\varepsilon E_3(v,s)$$

is considered, where $\varepsilon\ll 1$, v is the specific volume, and s is the entropy, then δ will no longer be an independent parameter and will be equal to M^2. In this case the instability state cannot be realized and the boundary between neutral stability and stability is given by (3.7).

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